Name:

SA402 · Dynamic and Stochastic Models Fall 2023 · Uhan

# Quiz 6 – 10/16/2023

**Instructions.** You have 30 minutes to complete this quiz. You may use your plebe-issue TI-36X Pro calculator. You may not use any other materials.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

| Problem | Weight | Score |
|---|---|---|
| 1a | 1 | |
| 1b | 1 | |
| 1c | 1 | |
| 2 | 1 | |
| 3a | 1 | |
| 3b | 1 | |
| 3c | 1 | |
| Total | | / 70 |

**Problem 1.** The Markov Company has a manufacturing cell that processes jobs during a 12-hour shift starting at 6 a.m. and ending at 6 p.m. Jobs leave the cell according to a Poisson process with rate $\lambda = 8$ per hour.

a. If the cell has processed exactly 10 jobs by 8 a.m., what is the probability that the cell will have processed strictly more than 30 jobs by 10 a.m.?

b. What is the probability that the cell will have processed its 50th job at or before 12 p.m.?

c. When are the first 4 jobs expected to be completed? (Assume all jobs are available starting at 6 a.m.)

**Problem 2.** Consider the arrival of customers to a doughnut shop in Downtown Annapolis. Does the independent increments property hold? Why or why not? Briefly explain.

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**Problem 3.** Patients arrive at a hospital emergency room at a rate of 2 per hour. A doctor works a 12-hour shift from 6 a.m. until 6 p.m. Assume the arrivals occur as a Poisson process.

a. Of patients admitted to the emergency room, 14% are classified as "urgent". What is the probability that the doctor will see 6 or fewer urgent patients during her shift?

b. What is the expected number of urgent patients between 9 a.m. and 3 p.m.?

c. The hospital also has a walk-in clinic to handle minor problems. Patients arrive at this clinic at a rate of 4 per hour. What is the probability that the total number of patients arriving at both the emergency room and walk-in clinic from 6 a.m. to 12 noon will be 30 or more?

| | $X \sim \text{Poisson}(\mu)$ | $X \sim \text{Exponential}(\lambda)$ | $X \sim \text{Erlang}(n, \lambda)$ |
|---|---|---|---|
| pmf / pdf | $p_X(a) = \begin{cases} \frac{e^{-\mu}\mu^a}{a!} & \text{if } a = 0, 1, 2, \ldots \\ 0 & \text{o/w} \end{cases}$ | $f_X(a) = \begin{cases} \lambda e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$ | $f_X(a) = \begin{cases} \frac{\lambda(\lambda a)^{n-1}e^{-\lambda a}}{(n-1)!} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$ |
| cdf | $F_X(a) = \sum_{k=0}^{\lfloor a \rfloor} \frac{e^{-\mu}\mu^k}{k!}$ | $F_X(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$ | $F_X(a) = \begin{cases} 1 - \sum_{k=0}^{n-1} \frac{e^{-\lambda a}(\lambda a)^k}{k!} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$ |
| expected value | $E[X] = \mu$ | $E[X] = \frac{1}{\lambda}$ | $E[X] = \frac{n}{\lambda}$ |
| variance | $\text{Var}(x) = \mu$ | $\text{Var}(X) = \frac{1}{\lambda^2}$ | $\text{Var}(X) = \frac{n}{\lambda^2}$ |